POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-10A(ii) : FLUID MECHANICS

(Applied Mathematics)

(Spl. Paper)

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$
   a) State the theorem of Blasius for a fixed cylinder in a steady two-dimensional irrotational motion.
   b) Define stream function.
   c) Define source, sink and doublet.
   d) Find the expression for the angular velocity of a pair of vortices.
   e) What is meant by wave profile ? Find the equation of wave profile at any instant of time referred to a given origin.
   f) What is the difference between an ideal and real fluid ?
   g) Define boundary layer thickness. What is its significance ?
2. a) Deduce the vorticity equation for an incompressible viscous fluid.
   b) Show that a viscous liquid cannot move without dissipation of energy by viscosity unless it moves as it rigid. 5 + 5
3. a) Show that in a simple harmonic train of surface waves, energy crosses a fixed vertical plane perpendicular to the direction of propagation at an average rate equal to group velocity.
   b) Simple harmonic waves of wavelength λ propagates over the surface of deep water. Prove that at a point whose depth below the undisturbed surface is h, the pressure at the instants when the disturbed depth of the point is $h+\eta$ to the undisturbed pressure at the same point bears the ratio
   $\left(1+\frac{\eta}{h}e^{-2\pi h/\lambda}\right) : 1$,
   atmospheric pressure and surface tension being neglected. 5 + 5

4. a) What is meant by Karman Vortex street ? Discuss the motion of rectilinear vortices lying on such a street. Also deduce the condition of stability of Karman Vortex street.

   b) Find the necessary and sufficient conditions that vortex lines may be at right angles to the stream lines. 5 + 5

5. a) Find the expression for the velocity potential and the equation of stream lines for the irrotational motion of a non-viscous liquid at rest at infinity in which a sphere is moving with uniform velocity, the motion being symmetrical about *z*-axis.

   b) Show that when a sphere of radius ' a ' moves with uniform velocity U through a perfect incompressible infinite fluid, the acceleration of a particle of the fluid at (r, 0) is

   $$3U^2\left(\frac{a^3}{r^4}-\frac{a^6}{r^7}\right)$$ 5 + 5

6. a) State and prove Milne-Thomson circle theorem. Apply the theorem to find the complex potential of a uniform flow with velocity U along negative x-axis past a fixed circular cylinder.

   b) A circular cylinder is fixed across a stream of velocity U with a circulation k round the cylinder. Show that the maximum velocity in the liquid is $2U+\frac{k}{2\pi a}$, where 'a' is the radius of the cylinder.

   5 + 5

7. a) An infinite elliptic cylinder with semi axes a, b is rotating round its axes with angular velocity ω in an infinite liquid of density ρ which is at rest at infinity. Show that if the fluid is under the action of no force, the moment of the fluid pressure on the cylinder round the centre is

   $\frac{1}{8}\pi\rho c^4\frac{d\omega}{dt}$ where $c^2=a^2+b^2$.

   b) Prove that if two rigid surfaces of revolution one of which surrounds the other, are moving along their common axis with velocities U_1, U_2 and space between them filled with homogenous liquid, the momentum of the liquid is $M_2U_2-M_1U_1$, where M_1, M_2 are the masses of liquid which either surface would contain.

   5 + 5